On Synchronizing Automata

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The princess in the dungeon

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- You do not know which cave you are in.
- You have a complete map of the dungeon.

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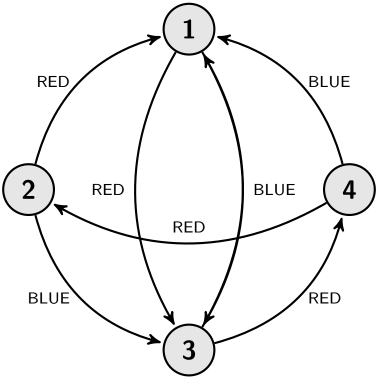

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- What is needed is a method that can guarantee that you end up in the same cave, no matter where you start.
- So you want to run a route that will always end up in the same spot
- Once you know your location you can go wherever you want (if everything is connected).

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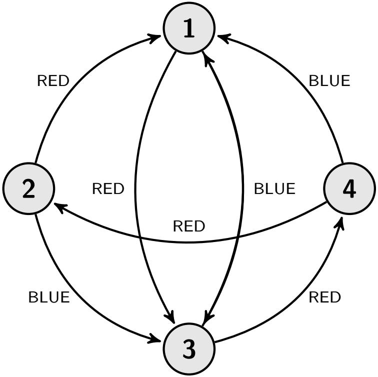

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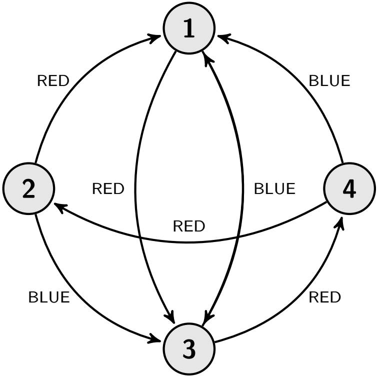

The magic words are BLUE RED BLUE BLUE.

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- The standard reaction to such errors can be considered versions of *backward recovery*.
- POWER OFF, REBOOT, RESTART FROM THE LAST SAVED CHECKPOINT.
- Such an option might not always exist or be ideal.

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- Before automatic work can be performed on them, the need to be aligned correctly.

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- Just like the princess in the the dungeon.

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If an automaton is synchronizing, it has infinitely many synchronizing words.

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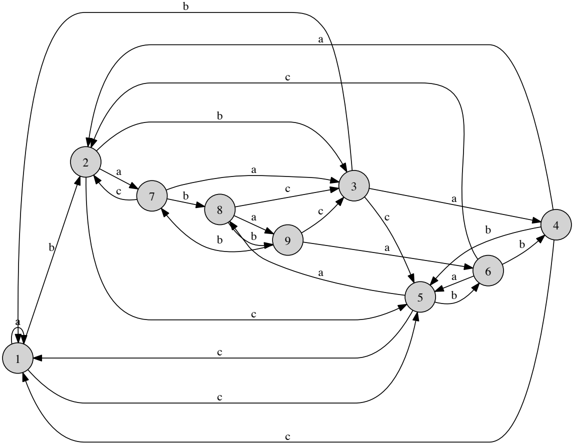
- **Černý Conjecture:** If an automaton is synchronizing and has n states, then it has a synchronizing word of at most $(n-1)^2$ letters.

Synchronizing?



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- Our questions then translate into: for a given set S of transformations on a finite set,
  1. does the semigroup generated by S generate a constant map?
  2. if so, can we bound the length of a word, on the generators, that gives a constant map?

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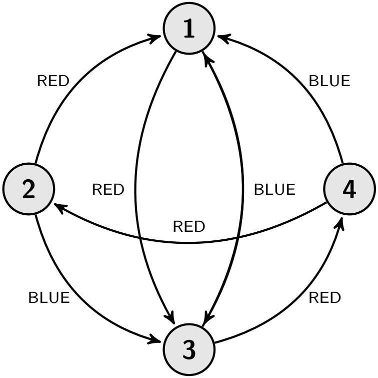

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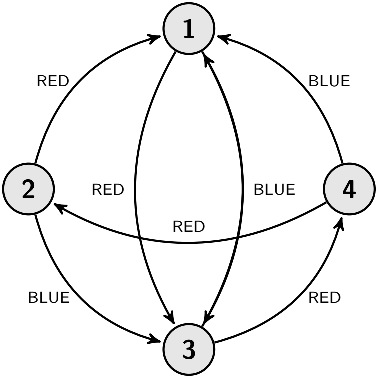

$$\text{BLUE} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 3 & 1 & 1 \end{pmatrix} \quad \text{RED} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}.$$

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- In our dungeon example there were four states $1, 2, 3, 4$ and we get:

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- If we compose those function from left to right in the order BLUE RED BLUE BLUE, we get

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Groups

Definition

Let S_n and T_n are, respectively, the symmetric group and the full transformation monoid on the set $X = \{1, \ldots, n\}$.
We say that a group $G \leq S_n$ *synchronizes* a transformation $t \in T_n \setminus S_n$ if the subsemigroup of T_n generated by $G \cup \{t\}$ contains a constant map.

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"Pure" groups theoretic definition: A permutation group G on X is synchronizing if no (proper, non-trivial) partition of X has a section (or transversal) S that is invariant under G (i.e. for which S^g is also a section for all $g \in G$)

Results on synchronizing groups

- Synchronizing groups must be *primitive* ([Araújo 2006], [Arnold and Steinberg 2006], [Neumann 2009]), that is, there is no (non-trivial and proper) G-invariant partition of X.

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- Synchronizing groups must be *primitive* ([Araújo 2006], [Arnold and Steinberg 2006], [Neumann 2009]), that is, there is no (non-trivial and proper) G-invariant partition of X.
- If the group G is primitive, but not synchronizing, let t be a transformation not synchronized by G with minimal rank. Then t has *uniform kernel*, i.e. all kernel classes are of the same size ([Neumann 2009]).

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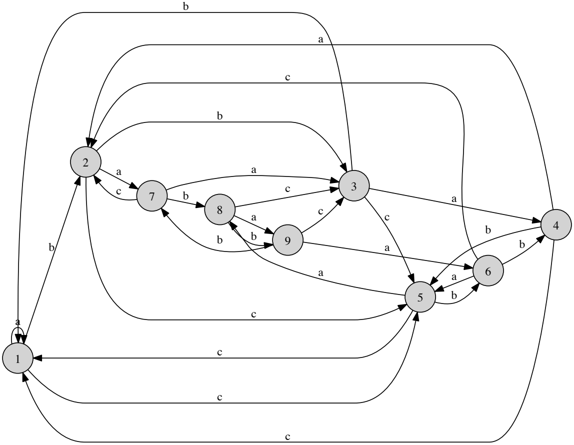
We have developed two different methods to recognize almost synchronizing groups. Both give an infinite list of examples (with little overlap).

Shortly back to our example



- $$\begin{aligned} a &= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 7 & 4 & 2 & 8 & 5 & 3 & 9 & 6 \end{pmatrix} \\ b &= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 1 & 5 & 6 & 4 & 8 & 9 & 7 \end{pmatrix} \\ c &= \begin{pmatrix} \{1,2,3\} & \{4,5\} & \{6,7\} & \{8,9\} \\ 5 & 1 & 2 & 3 \end{pmatrix} \end{aligned}$$

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- If our conjecture is right, we have a synchronizing word.

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- Let $T \subset X$ be a clique of $\overline{\Gamma}_S$ and define $m(S, T)$ to be the smallest number m such that the intersection of the closed neighborhoods of *any* m points of T in $\overline{\Gamma}_S$ is equal to T.

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- Finally, consider all uniform transformations t that are not synchronized by G. Let $m(G)$ be the maximal $m(S, T)$ where S is the image of t and T a block in the kernel of t.
- $m(G)$ is the *non-synchronizing parameter* of G.

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- Let $M(P, P') = \frac{|P \cap P'|}{k}$.
- Set $M(G)$ to be the maximum over all such pairs.

Main Theorem

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Let $G \leq S_n$ be a non-synchronizing group with $m(G) = 2$ and $M(G) \leq 1/2$. If $t \in T_n$ is a transformation such that $Ker(t)$ is a non-uniform partition, then $\langle G, t \rangle$ contains a constant function; that is, G is almost synchronizing.

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We do not know any non-synchronizing G with $m(G) = 2$ and $M(G) > 1/2$.

Almost non-synchronizing

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- A primitive permutation group G with the property that the only G-invariant graphs X with coinciding clique and chromatic numbers are pseudo cores, is almost synchronizing.
- *Pseudo cores*: every endomorphism of X is either an automorphism or a colouring.
- We also obtained infinite families of examples for this method.

Examples of almost-synchronizing groups

- The symmetric group S_m acting on 2-sets for m even, $m \geq 6$.
- G a subgroup of $\mathrm{P\Gamma L}(n, q)$ containing $\mathrm{PSL}(n, q)$ with $n \geq 5$, acting on the lines of the projective space.
- $G$the group $\mathrm{P\Gamma L}(3, q) \cdot 2$ or a subgroup containing $\mathrm{PSL}(3, q) \cdot 2$, where 2 denotes the *inverse transpose automorphism*, acting on the set of incident point-line pairs in the projective plane (where the outer automorphism induces a duality of the plane interchanging points and lines).

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- For each $k \geq 2$, classify the primitive almost synchronizing groups such that $m(G) = k$.

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- For each $k \geq 2$, classify the primitive almost synchronizing groups such that $m(G) = k$.
- Is it true that primitive groups are almost synchronizing?

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